Ch 10 Exponential and Logarithmic Functions

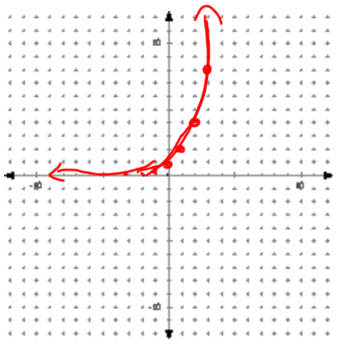
10-1 Exponential Functions

Exponential function--function with a variable in the exponent

ex

$y = 2^x$

x	y
-3	$\frac{1}{2^3} = \frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4



Form

$y = a\,b^x$

$a \neq 0$

$b > 0$

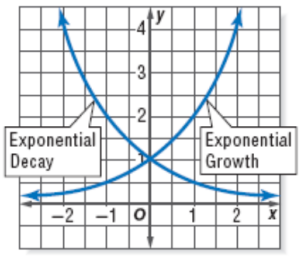
$b \neq 1$

graph on calculator

Characteristics (p524)

1. The function is continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x-axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y-intercept is a.
6. The graphs of $y = ab^x$ and $y = a\left(\frac{1}{b}\right)^x$ are reflections across the y-axis.

There are two types of exponential functions: **exponential growth** and **exponential decay**. The base of an exponential growth function is a number greater than one. The base of an exponential decay function is a number between 0 and 1.



Key Concept — *Exponential Growth and Decay*

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

FARMING In 1983, there were 102,000 farms in Minnesota, but by 1998, this number had dropped to 80,000.

a. Write an exponential function of the form $y = ab^x$ that could be used to model the farm population y of Minnesota. Write the function in terms of x, the number of years since 1983.

$y = ab^x$

a = original amount
b = rate of change
y = new amount
x = time

$80{,}000 = 102{,}000\, b^{15}$

$(.78)^{\frac{1}{15}} = (b^{15})^{\frac{1}{15}}$

$.98 = b$

$y = 102{,}000(.98)^x$

b. Suppose the number of farms in Minnesota continues to decline at the same rate. Estimate the number of farms in 2010.

$y = 102{,}000(.98)^x$

$y = 59{,}115$

$59{,}116$

In December of 1990, there were 5,283,000 cell phones in the U.S.
In December of 2000, there were 109,478,000.

Write an equation in exponential form.
Find the number of cell phones in 2010.

$y = ab^x$

$109{,}478{,}000 = 5{,}283{,}000\, b^{10}$

$1.35 = b$

$y = 5{,}283{,}000(1.35)^x$

$2{,}136{,}000{,}000$

Solving exponential equations.

ex

$8^x = \frac{1}{4}$

$2^{3x} = 2^{-2}$

$3x = -2$

$x = -\frac{2}{3}$ ✓

ex

$5^{4-t} = 25^{t-1}$

$(5^2)^{t-1}$

$4 - t = 2t - 2$

$6 = 3t$

$2 = t$ ✓

ex

$\sqrt[3]{32} = 2^x$

2^5

$2^{\frac{5}{3}} = 2^x$

$\frac{5}{3} = x$

ex

$9^{x-3} = 27$

$3^{2(x-3)} = 3^3$

$2x - 6 = 3$

$2x = 9$

$x = 4.5$

Do:

1. $4^{x-2} = 64^{x}$

4^{3x}

$x - 2 = 3x$

$-2 = 2x$

$-1 = x$

2. $4^{2x+5} = 16^{x+1}$

(4^2)

$2x + 5 = 2x + 2$

$\varnothing$

Exponential Inequalities

ex

$3^x < \frac{1}{27}$

$3^x < 3^{-3}$

$x < -3$

ex

$5^x > \sqrt{125}$

5^3

$5^x > 5^{3/2}$

$x > \frac{3}{2}$

ex

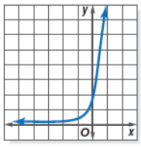
$25^{2x} < 5^{(x+6)}$

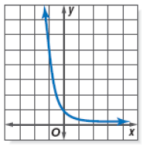
$x < 2$

1. **OPEN ENDED** Give an example of a value of b for which $y = b^x$ represents exponential decay.
2. **Identify** each function as *linear*, *quadratic*, or *exponential*.
   a. $y = 3x^2$ b. $y = 4(3)^x$ c. $y = 2x + 4$ d. $y = 4(0.2)^x + 1$

Match each function with its graph.

3. $y = 5^x$
4. $y = 2(5)^x$
5. $y = \left(\frac{1}{5}\right)^x$

a. 

b. 

c. 

ANIMAL CONTROL **For Exercises 19 and 20, use the following information.**
During the 19th century, rabbits were brought to Australia. Since the rabbits had no natural enemies on that continent, their population increased rapidly. Suppose there were 65,000 rabbits in Australia in 1865 and 2,500,000 in 1867.

19. Write an exponential function that could be used to model the rabbit population y in Australia. Write the function in terms of x, the number of years since 1865.
20. Assume that the rabbit population continued to grow at that rate. Estimate the Australian rabbit population in 1872.

HW
p528-529
21, 25, 27-29, 33, 35, 39-53odd,
57-60(look at picture)